Applications of Fresnel-Kirchhoff diffraction theory in the analysis of human-motion Doppler sonar grams

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Abstract: Observed human-gait features in Doppler sonar grams are explained by using the Boulic-Thalmann (BT) model to predict joint angle time histories and the temporal displacements of the body center of mass. Body segments are represented as ellipsoids. Temporally dependent velocities at the proximal and distal end of key body segments are determined from BT. Doppler sonar grams are computed by mapping velocity-time dependent spectral acoustic-cross sections for the body segments onto time-velocity space, mimicking the Short Time Fourier Transform used in the Doppler sonar processing. Comparisons to measured data indicate that dominant returns come from trunk, thigh and lower leg.

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1. Introduction

Doppler sonar grams provide a wealth of information about human gait, e.g., Ekimov and Sabatier (2009). Individual body parts have different acoustic cross-sections and velocities resulting in unique Doppler signatures. Similar signatures are measured by Doppler radar systems, e.g., Zhang *et al.* (2007). This is not surprising since the sonars and radars operate at comparable wavelengths.

In this paper the observed features in Doppler sonar grams are explained using the following approach. The human body is represented as a segmented link system following Winter (2009). The Boulic-Thalmann (Boulic *et al.*, 1990) model is used to predict joint angle time histories and the temporal displacements of the body center of mass. The velocity at the proximal and distal end of each key body segment as a function of time is determined from the Boulic-Thalmann (BT) joint rotations and body translations using simple rigid body physics as described in Bradley (2009). Scattering is assumed to occur from seven types of body segments: the foot, lower leg, thigh, trunk, head-neck, upper arm and lower arm-hand. For the purposes of scattering, the body segments are modeled as ellipsoids. The dimensions of these ellipsoids are estimated from the segment length, mass and density. At-rest acoustic scattering cross-sections for the segments are determined using Fresnel-Kirchhoff diffraction theory. Velocity and time dependent spectral acoustic-cross sections are obtained by exploiting the fact that the velocity at points on a rotating-translating rigid body must continuously vary across its length. Doppler sonar grams are computed by mapping the time dependent spectral acoustic-cross sections of

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14. ABSTRACT
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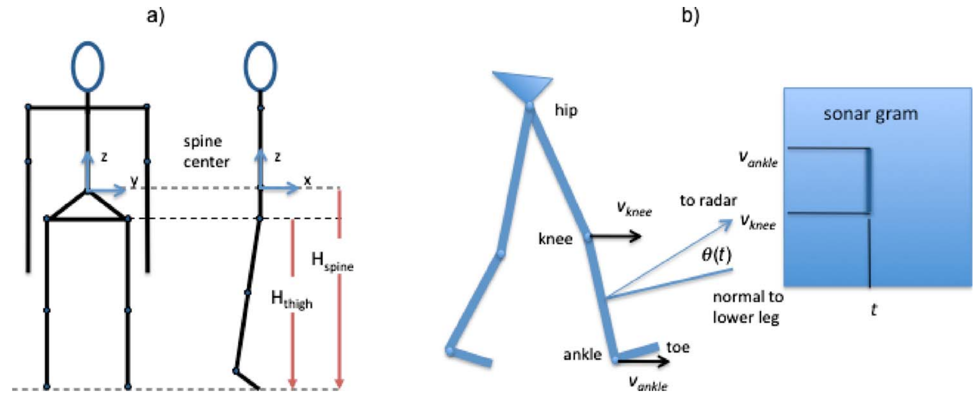

Fig. 1. (Color online) (a) The coordinate system used in the Boulic-Thalmann model. (b) Computation of motion dependent effects of scattering from the lower leg.

the various body segments onto time-velocity space. This mapping is implemented in a way that mimics the Short Time Fourier Transform (STFT) used in the processing of Doppler sonar data.

The Boulic-Thalmann model is an empirical model that describes the gait of an average human being. It was originally developed to provide a computationally efficient tool for aiding computer animation. Recently it has been found to be useful in explaining the features seen in Doppler sonar and radar grams.

A walk cycle consists of two steps. It begins with left foot heel strike (HS), continues through right foot HS, left foot toe off (TO) and ends with the second HS of the left foot. The stride period is the time required to take two steps. It can be related to the average velocity of walking v and the body height H via Inman's law: $T(v,H)=0.980(H/v)^{1/2}$. For a constant speed of advance, Boulic *et al.* (1990) define the relative time to be $\tau = t/T$. In one complete walk cycle the relative time will vary between 0 and 1.

In the BT model, the body is represented as a segmented link system. The coordinate system used in the BT model is shown in Fig. 1(a). The center of the coordinate system is referred to as the spine center. The z-axis is in the vertical direction and the x-axis points in the direction the individual is walking. The lower body is free to rotate in three directions (roll, pitch and yaw) about the spine center. The torso is free to rotate (yaw) about the spine center independently of the lower body. Flexion occurs at the hip, knee and ankle for the right and left legs. Flexion also occurs at the shoulder and elbow for the right and left arms. Left and right side leg and arm motions are symmetrical. Additionally, the body translates about spine center. In total there are twelve degrees of freedom.

This paper assumes a measurement geometry in which an individual is walking directly toward the sonar. This geometry maximizes the Doppler sonar's response to sagittal plane swinging motions of the legs and arms as well as sagittal plane motions of the trunk.

2. Body at rest

For the purposes of estimating scattering, the body parts are represented by rigid ellipsoids with Cartesian coordinate representation defined by Eq. (1). For the foot, lower leg, thigh, upper arm, forearm-hand and head-neck (see Table 1), it is assumed that $b=a$ with $a<c$. The assumption $b=a$ produces prolate spheroids. For the trunk it is assumed that $b=2a$ in order to represent its greater width/depth ratio.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1. \tag{1}$$

In all cases, the value of the parameter c is one half the segment length from Table 1. The volume of a segment is $V=\frac{4}{3}\pi abc$. The parameter a is determined from the relationship M

Table 1. Anthropometric data (Winter, 2009). H is height in m and M is mass in kg.

Body part	Proximal/distal	Length	Mass	Density (gm/cm^3)
Thigh	Hip/knee	$0.245H$	$0.100M$	1.05
Lower leg	Knee/ankle	$0.285H$	$0.0465M$	1.09
Foot	Ankle/toe tip	$0.152H$	$0.0145M$	1.10
Upper arm	Shoulder/elbow	$0.188H$	$0.028M$	1.07
Forearm and hand	Elbow/finger tip	$0.253H$	$0.022M$	1.14
Trunk	Spine center/throat bot	$0.288H$	$0.497M$	1.03
Head and neck	Throat bot/head top	$0.259H$	$0.081M$	1.11

$=\rho V$ where M and ρ are the segment mass and density as defined in Table 1. For the case of the prolate spheroid ($b=a$), this results in $a=[M/(\frac{4}{3}\pi\rho c)]^{1/2}$. For the trunk ($b=2a$), the result is $a=[M/(\frac{8}{3}\pi\rho c)]^{1/2}$.

Fresnel-Kirchhoff diffraction theory can be used to estimate the amplitude of a wave scattered by a smooth, rigid surface [Elmore and Heald, 1985; National Defense Research Committee (NDRC), 1945]. Smooth in this context means the two local radii of curvature of the surface are always large compared to the incident sound wavelength. The amplitude of the sound scattered back from a smooth target to the sonar is

$$p(P_s) = -i\frac{A}{\lambda}e^{-2\pi i f t}\int_S \frac{\cos\phi}{r^2}e^{2ikr}dS, \tag{2}$$

where λ is the wavelength of sound, f is the frequency, k is acoustic wavenumber, r is the distance from a point on the scattering surface S to the sonar location P_s, the quantity A is proportional to the amplitude of the radiated sound from the sonar and ϕ is the angle between the local normal to the surface S and the incident (and backscattered) sound. The acoustic scattering cross section σ defined by

$$\sigma(P_s) = \left|\frac{1}{\lambda}\int_S \cos\phi e^{2ikr}dS\right|^2, \tag{3}$$

has dimensions of area and is a measure of the ability of an object to scatter sound back to a receiver that is in the same location as a transmitter (Clay and Medwin, 1977). The scattering cross section for an ellipsoid defined by Eq. (1) is (Blake, 1991)

$$\sigma(\theta) = \frac{\pi a^2 b^2 c^2}{(a^2\sin^2\theta + c^2\cos^2\theta)^{1/2}}, \tag{4}$$

where the scattering angle θ is measured relative to the z-axis in the xz (sagittal) plane and the dimensions of the ellipsoid are assumed to be large in comparison with the wavelength λ. For one of the prolate spheroidal shaped body segments at normal (broadside) incidence, $\theta=\pi/2$ and the scattering cross section is $\sigma(\pi/2)=\pi c^2$. When the body segment is parallel to the direction to the sonar, $\theta=0$ and $\sigma(0)=\pi a^4/c^2$. The ratio of normal incidence to parallel incidence scattering cross sections is $\sigma(\pi/2)/\sigma(0)=(c/a)^4$.

3. Motion dependent effects

Equations (2)–(4) apply only to a body at rest. To model the motion dependent effects of scattering from a body segment at a particular time instance t, the velocities v_p and v_d at the proximal end and distal ends of the segments and the scattering angle $\theta(t)$ are computed using the BT model. This is illustrated in Fig. 2 for the case of the lower leg. A linear variation in velocity

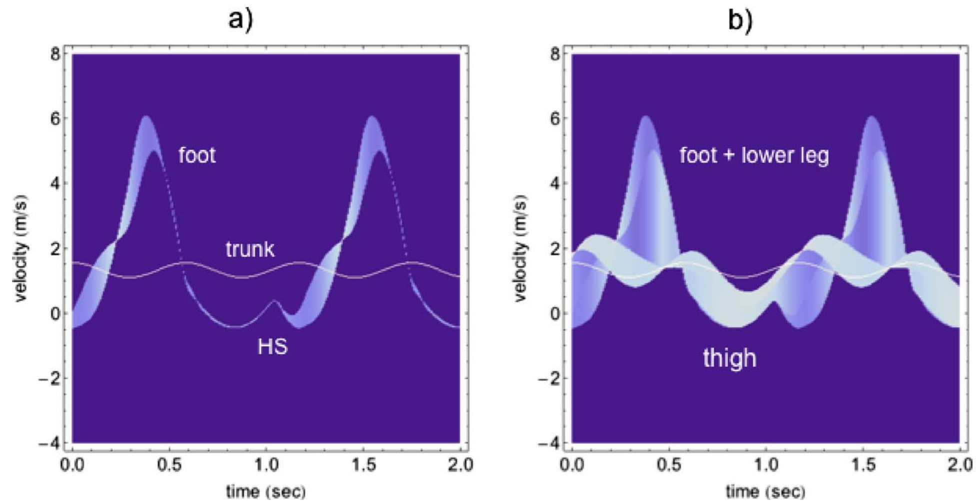

Fig. 2. (Color online) (a) Contributions to a Doppler sonar gram from the right foot and trunk of an individual walking toward the sonar. (b) Additional contributions to a Doppler sonar gram from the lower right leg and thigh of the same individual.

across the length of the segment is assumed. The segments contribution to a Doppler sonar gram at time t in the velocity band (v_1, v_2) within the range of proximal and distal velocities is assumed to be proportional to the spectral acoustic cross section

$$\sigma(v_1,v_2,t) = \frac{v_2 - v_1}{|v_d - v_p|}\sigma[\theta(t)]. \qquad (5)$$

If $v_p = v_d$ then all acoustic energy scatters at the same velocity (frequency) and the contribution to the Doppler sonar gram proportional to $\sigma(v_d, v_d, t) = \sigma[\theta(t)]$.

Figure 2(a) shows the contribution to a Doppler sonar gram from the right foot for an individual 1.89 m tall walking toward the sonar at a speed of 1.34 m/s. Heel strike (HS) of the left foot occurs at about 1.1 s. At this time both feet are in contact with the ground and the velocity is zero. Following left HS, the right foot swings forward producing the wide range of velocities seen between 1.25 and 1.5 s. During this time interval there are substantial differences between the velocity of the ankle and the toe. These differences produce the velocity spreads at individual times in the gram in accordance with Eq. (5). Also shown in Fig. 2(a) is the contribution of the trunk. The trunk contribution to the gram occurs in a very narrow velocity band due to the small differences between the velocity of the spine center and base of neck (proximal and distal segments for trunk). This velocity oscillation is caused by accelerations of the body center of mass resulting from two heel strikes and toe offs and is twice the gait frequency.

Figure 2(b) shows the additional contributions to the Doppler sonar gram from the right lower leg and thigh. The contribution to the gram from the thigh occurs at low velocities similar to those of the trunk but with a broader overall spread and in velocity and weaker energy levels due to the smaller relative size of the thigh in comparison to the trunk. The return from the lower leg fills in the region between the return from the foot and thigh. Its amplitude is stronger than that of the foot and is most pronounced when the lower leg is perpendicular to the ground. At these times the lower leg presents maximum acoustic cross section to the sonar. The trunk by way of contrast always presents maximum acoustic cross section to the sonar provided that the individual is walking directly toward or away from the sonar.

The contribution to the gram from all body segments is shown in Fig. 3(a). The quiet period between 0.5 and 1.0 s [see Fig. 2(a)] is now filled in by the symmetrical motion of the left

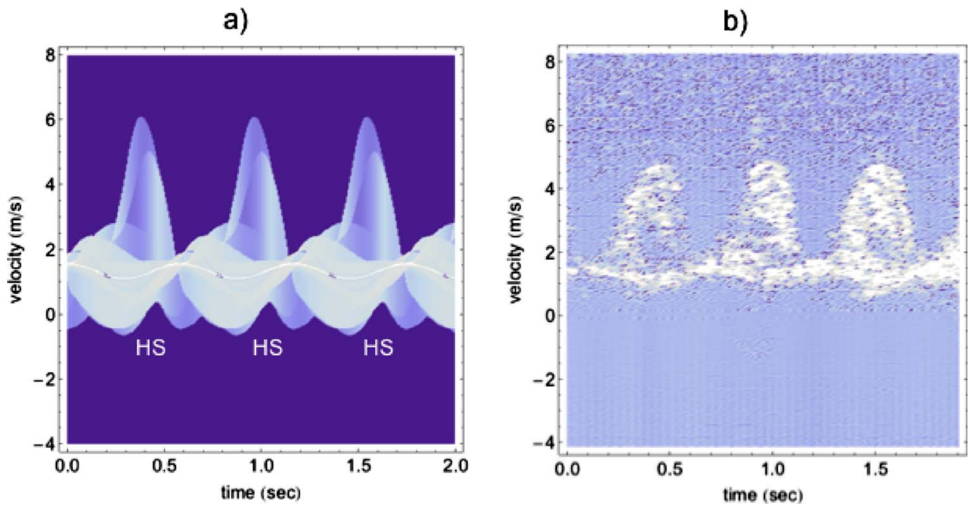

Fig. 3. (Color online) (a) Contributions to a Doppler sonar gram from all body segments of an individual walking toward the sonar. (b) Measured data for a scenario similar to that modeled in Fig. 3(a). The height of the individual is 1.89 m and the speed of walking is 1.34 m/s.

thigh, lower leg and foot. The contributions from the head-neck together with the right and left side upper arm and forearm-hand are represented in Fig. 3(a) as well. The head-neck contribution occurs in the same time-velocity range as the trunk but with weaker amplitude. The returns from the arms are smaller than those from the legs. Figure 3(b) shows a measurement made under conditions similar to those for which the model predictions were made (Ekimov and Sabatier, 2009). The dynamic range in the measurement is about 20 dB. The model predictions have more dynamic range: 50 dB. The trunk oscillations are clearly visible in the measured data as are the three rapid leg swings. Weaker amplitude returns near zero velocity in the measured Doppler sonar grams are only faintly visible. The returns from the legs in the measured gram are somewhat broader and with about 20% lower peak velocity than predicted by the model. There are two explanations for this difference. First, the sonar may not be seeing the return from the foot. A second more likely reason is that the BT model may not be accurately predicting the rotation of the hip. BT assumes a near sinusoidal variation in hip angle through the course of a walk cycle. The data indicate that this individual accelerates his hip quicker from toe off than BT but with less peak velocity. The individual then slows down quicker to heel strike than BT. This explanation is consistent with the activity of the quadriceps and hamstring muscles during the walk cycle (Inman *et al.*, 1981).

In general, the model and data show that the dominant returns in the gram come from the narrow velocity-large amplitude return of the trunk plus returns from the thigh and lower leg. Higher resolution systems should be able to see the returns from the foot and an absence of return at the times of heel strike.

4. Summary

Fresnel-Kirchhoff diffraction theory has been applied to the analysis of the time-frequency structure observed in human motion Doppler sonar grams. The body is represented as a segmented link system. Scattering is assumed to occur from the thigh, lower leg, foot, upper arm, forearm and hand, trunk, head and neck. The Boulic-Thalmann model is used to model motion effects. Comparisons have been made to measured data. The model and data show that the dominant returns in a micro Doppler sonar gram come from the human trunk, thigh and lower leg.

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